

Mathematical Modeling of Unsteady Flow for AL- Kahlaa Regulator River

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Abstract: - The study of the river flow predictions in open channels is an important issue in hydrology and hydraulics . Consequently, this paper is concerned with studying the unsteady flow that may exist in open channel , and its mathematically governed by the Saint Venant equation using a four-point implicit finite difference scheme.

From many hydrologic software, HEC-RAS (Hydrologic Engineering Center – River Analysis System) is a good choice to develop the hydraulic model of a given river system in the south of Iraq represented by Al-Kahlaa River by a network of main channel and three reach and a total of 57 cross sections with 3 boundary sections for one of the applications . The model is calibrated using the observed weekly stage and flow data . The results show that a good agreement is achieved between the model predicted and the observed data using the values Manning's(n=0.04) for over bank with the values of Manning's (n=0.027) for main channel and also with using time weighting factor (θ) equal one . Lastly , the AL- Kahlaa River HEC-RAS model has been applied to analyze flows of Al Huwayza marsh feeding rivers (Al Kahlaa River and its main branches) , evaluation of their hydraulic performance under two hydraulic model scenarios .The results demonstrate that in case of high flow discharge it is found that cross sections flooded and inadequate for such flows. While, flows are remained within cross section extents during drought season .

Keywords: *Mathematical model, St. Venant equations, Finite difference methods, HEC-RAS*

I. INTRODUCTION

Common examples of unsteady open channel flows include flood flows in rivers and tidal flows in estuaries , irrigation channels , headrace channel of hydropower plants , navigation canals , storm water systems and spillway operation .In unsteady open channel flows , the velocities and water depths change with time and longitudinal position . For one-dimensional applications, the relevant flow parameters (e.g. V and d) are functions of time and longitudinal position . Analytical solutions of the basic equations are nearly impossible because of their non- linearity , but numerical techniques may provide approximate solutions [6]. The equations of unsteady channel flow (Saint-Venant equations) formalize the main concepts and hypotheses used in the mathematical modeling of flow problems, and are formulated in the 19th century by two mathematicians , de Saint Venant and Bousinesque as early as in 1871. With the computer revolution in twentieth century made a new era where numeric methods can be utilized effectively to solve nonlinear partial differential equations. This study is provided a description of the methodology is used to develop the hydraulic model for the Al Kahlaa river . The hydraulic model is performed with using the one-dimensional U.S.Army Corps of Engineers Hydrologic Engineering Center River Analysis System , HEC-RAS version 4.1.0. . The Al Kahlaa HEC-RAS model is developed to characterize flows at all study sites , extending from the its head regulator to the downstream model boundary at the Al Huwayza Marsh .The hydraulic model is developed for the main channel, and the overflow channel .

II. GOVERNING EQUATIONS

The model is based on the one-dimensional unsteady flow equations (Saint-Venant equations) , consisting of the equations for the conservation of mass (continuity),and conservation of momentum. There are other ways of representing the Saint-Venant equations which are based upon the same assumptions but are expressed in terms of different set of dependent variables , depending on the assumptions used in their derivations [2] , these equations can be written as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \mp q \quad \dots\dots(1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{\beta \cdot Q^2}{A} \right) + g \cdot A \cdot \left(\frac{\partial h}{\partial x} + S_f \right) = 0 \quad \dots(2)$$

where: A = cross – sectional area of flow, (L²) ; β: momentum correction factor ; g = acceleration due to gravity constant, (L / T²) ; Q = discharge (through A), (L³ / T) ; x = longitudinal distance along the channel or river, (L) ; t = time, (T) ; h = is the surface elevation , (L) ; and S_f = friction slope.

III. NUMERICAL SOLUTION METHODS

The solution of (Saint-Venant equations) by analytical methods ,as already indicated, has been obtained only for simplified and restricted cases , but numerical techniques may provide approximate solutions [10].

For the purposes of this study , easier solution for solving the complete one dimensional St.Venant equations of unsteady flow ,can be obtained by using Finite Difference Method , which is the most appropriate and less complicated in 1D approach in comparison with other methods . Finite- difference schemes can be classified the explicit scheme and the implicit scheme. In the explicit scheme, the finite difference equations are usually linear algebraic equations from which the unknowns can be evaluated explicitly a few at a time. The unknowns occur implicitly in the finite difference equations of implicit scheme, which are usually nonlinear algebraic equations [1]. The implicit method is mathematically more complicated , but it is stable, efficient and much faster than the explicit method , especially when treating long term phenomena such as floods in major river. Moreover ,the implicit method can also handle channel geometry varying significantly from one channel cross section to the next [2].

IV. PREISSMANN IMPLICIT SCHEME

Of the various implicit difference schemes which have been developed , the " weighted four- point " schemes appear most advantageous and stable because it of its simple structure with both flow and geometrical variable in each grid point[2] .

By using four-point implicit finite difference scheme as shown in **Figure 1**, the value of function $f_p (x , t)$ at point P , which is located in the distance interval $i, i+1$ and time interval $j, j+1$ is approximated as follows:

$$f_p = \theta \cdot \psi \cdot f_{i+1}^{j+1} + \theta \cdot (1 - \psi) \cdot f_i^{j+1} + (1 - \theta) \cdot \psi \cdot f_{i+1}^j + (1 - \theta) \cdot (1 - \psi) \cdot f_i^j \quad \dots\dots (3)$$

- θ: weight factor for the time t (0 ≤ θ ≤ 1),
- ψ: weight factor for the distance x (0 ≤ ψ ≤ 1),
- j: index of time level,
- i: index of cross- section.

The spatial derivatives of the function f at point (P) are given by :

$$\frac{\partial f}{\partial x} \Big|_P \approx \frac{\theta (f_{i+1}^{j+1} - f_i^{j+1}) + (1 - \theta) (f_{i+1}^j - f_i^j)}{\Delta x} \quad \dots\dots (4)$$

And the temporal derivatives of the function f becomes:

$$\frac{\partial f}{\partial t} \Big|_P \approx \frac{\psi (f_{i+1}^{j+1} - f_{i+1}^j) + (1 - \psi) (f_i^{j+1} - f_i^j)}{\Delta t} \quad \dots\dots (5)$$

And variables other than derivatives are approximated in a similar manner, i.e.

$$f_p \approx \frac{\theta (f_i^{j+1} + f_{i+1}^{j+1}) + (1 - \theta) (f_i^j + f_{i+1}^j)}{2} \quad \dots\dots (6)$$

Clearly, a whole family of finite difference schemes may be obtained by varying the parameters ψ and θ , all such schemes, however, are four- point schemes.

The four-point difference scheme becomes implicit for all values of θ greater than zero [4]. When $\psi = 0.5$,the system of “Equation (3)” to “Equation (5)” becomes the Preissmann four- point scheme, so point P can move along t-axis only , in away controlled by the weighting parameter θ [9] .

whereby , the temporal derivatives become :

$$\frac{\partial f}{\partial t} \approx \frac{(f_{i+1}^{j+1} - f_{i+1}^j) + (f_i^{j+1} - f_i^j)}{2\Delta t} \dots\dots\dots (7)$$

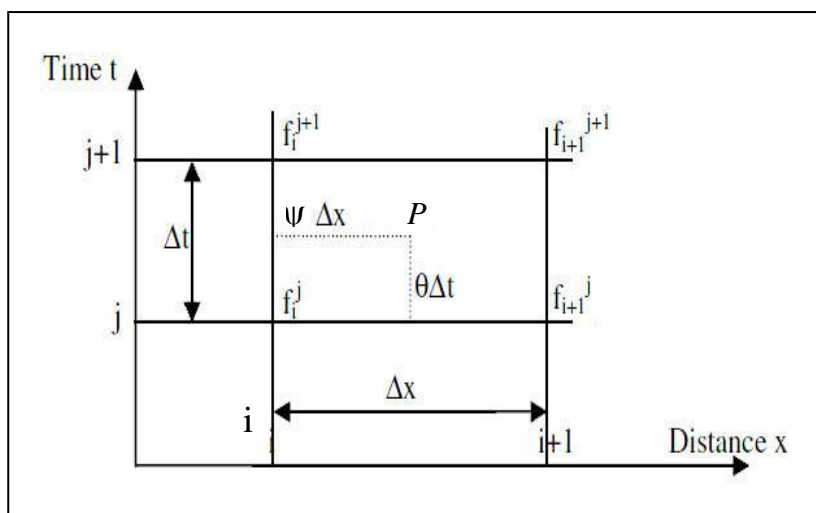


Figure (1) Four –point difference scheme

The finite – difference form of the continuity equation (F_i) and momentum , (G_i) is produced by applying these numerical approximations to “Equation (1)” and “Equation (2)” yields:

$$F_i = \frac{1}{\Delta x_i} [\theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1 - \theta)(Q_{i+1}^j - Q_i^j)] + \frac{1}{2\Delta t_j} [(A_{i+1}^{j+1} - A_{i+1}^j) + (A_i^{j+1} - A_i^j)] - \frac{1}{2} [\theta(q_i^{j+1} + q_{i+1}^{j+1}) + (1 - \theta)(q_i^j + q_{i+1}^j)] = 0 \dots\dots\dots (8)$$

Similarly, the finite difference form of the momentum equation (G_i) is written as:

$$G_i = \frac{1}{2\Delta t_j} [(Q_i^{j+1} - Q_i^j) + (Q_{i+1}^{j+1} - Q_{i+1}^j)] + \frac{1}{\Delta x_i} \left\{ \theta \left[\left(\frac{\beta \cdot Q^2}{A} \right)_{i+1}^{j+1} - \left(\frac{\beta \cdot Q^2}{A} \right)_i^{j+1} \right] + (1 - \theta) \left[\left(\frac{\beta \cdot Q^2}{A} \right)_{i+1}^j - \left(\frac{\beta \cdot Q^2}{A} \right)_i^j \right] \right\} + \frac{g \cdot A_p}{\Delta x_i} [\theta(h_{i+1}^{j+1} - h_i^{j+1}) + (1 - \theta)(h_{i+1}^j - h_i^j)] + \frac{g \cdot A_p}{2} \left\{ \theta [(S_f)_{i+1}^{j+1} + (S_f)_i^{j+1}] + (1 - \theta)[(S_f)_{i+1}^j + (S_f)_i^j] \right\} = 0 \dots\dots\dots (9)$$

where the friction slope S_f is computed from Manning 's equation for uniform flow:-

$$S_{fi} = \frac{n^2 |Q_i| Q_i}{A_i^2 R_i^{4/3}} \dots\dots\dots (10)$$

Equation (8)” and “Equation (9)” form a system of two algebraic equations which are nonlinear with respect to unknowns, the values of h and Q at the net points (i , j+1) and (i+1 , j+1). The terms associated with the net points (i , j) and (i+1 , j) are known from either the initial conditions or previous computations.

The two equations contain only four unknowns : Thus, . Thus , these equations cannot be solved for the $(Q_i^{j+1}), (h_i^{j+1}), (Q_{i+1}^{j+1}), (h_{i+1}^{j+1})$ unknowns since there are two more unknowns than equations . But, when applied these equations(F_i , G_i) to the (N-1) reaches between the upstream and downstream boundaries of the river system with N cross sections , a total of (2N-2) equations with 2N unknowns is obtained (N denotes the total number of nodes or cross sections). The two supplementary equations needed to close the system are provided from upstream and downstream boundary conditions. The resulting system of 2N equations with 2N unknowns [4].

The system of nonlinear equations can be expressed in functional form in terms of the unknowns h and Q at time level j+1 , as follows:

$$G_0(h_1, Q_1) = 0 \leftarrow \text{Upstream boundary condition}$$

$$\left. \begin{aligned} F_1(h_1, Q_1, h_2, Q_2) &= 0 \\ G_1(h_1, Q_1, h_2, Q_2) &= 0 \\ \cdot \\ \cdot \\ F_i(h_i, Q_i, h_{i+1}, Q_{i+1}) &= 0 \\ G_i(h_i, Q_i, h_{i+1}, Q_{i+1}) &= 0 \\ \cdot \\ \cdot \\ F_{N-1}(h_{N-1}, Q_{N-1}, h_N, Q_N) &= 0 \\ G_{N-1}(h_{N-1}, Q_{N-1}, h_N, Q_N) &= 0 \end{aligned} \right\} \dots\dots\dots (11)$$

$$F_N(h_N, Q_N) = 0 \leftarrow \text{Downstream boundary condition}$$

The system (11) of nonlinear algebraic equations is solved by using the Newton-Raphson iterative method . The application of that method reduces the solving of the system of non-linear equations to the solving of the system of linear equations through the series of iterations. It should be noted that although the system of “Equation (11)” involves 2N unknowns, each equation contains a maximum of four unknowns. This can be used to a great advantage in the computational schemes [1].

In this technique , trial values are assigned to the 2N unknowns through the Kth trial. Substitution of these trial values in a system of “Equation (11)” usually results in the left- hand side of the equation being nonzero. This nonzero value is termed a residual , and the solution is obtain by adjusting the trail variable values until the residuals vanish.

Hydraulic models developed for this study cover Al Kahlaa river system as case study . This model is based on both continuity and momentum equations to solve unsteady flow problems.

The residuals and the partial derivatives of “Equation (11)” are related according to the Newton method resulting a system of 2N linear equations in 2N unknowns. Therefore , any of the standard methods for the solution of simultaneous , linear , algebraic equation can be applied , e.g., Gaussian elimination[4]

V. STUDY AREA AND DATA USED

The study for this paper is Al Kahlaa river which is one feeder of Al Huwayza Marsh. Al Huwayza feeders can be classified into two types depending on the existence of water control structures. The first is Al Msharah, and the second is Al Kahlaa Rivers which are controlled by a head regulators located upstream of each one , both Al Kahlaa and Al Msharah Rivers sharing the same intake located north of Al Amarah Barrage on Tigris River . **Figure 2** illustrates the location of Al Huwayza Marsh feeders .

A regulator has been constructed on the Al Kahlaa River, 1.35 km downstream from the Msharah off take. Hence , the water entering Al Kahlaa River is controlled by this head regulator, with a design discharge capacity of 400 m³/s and the operating discharge capacity of 300 m³/s .

Therefore , the length of the main Al Kahlaa River extending from its head regulator to the center of Al Kahlaa town is about 26 km , the main Al Kahlaa River is divided into three main rivers, as follows:

- Al Husachi River (Adil Jasim River) with a length of about 25 km. This river directly feeds Al Huwayza Marsh by a pipe overpass at its end.
- Um At Toos River (AshShina'shely) with a length of about 25km and directly feeds Al-Huwayza Marsh.
- Al Zubair River (Abu Kha'ssaf) with a length of about 22km. The water at the end of these branches is pumped to Al Ma'eel channel.

All data used in this study, which were provided by the Ministry of Water Resources, Center for Restoration of Iraqi Marshlands [8].

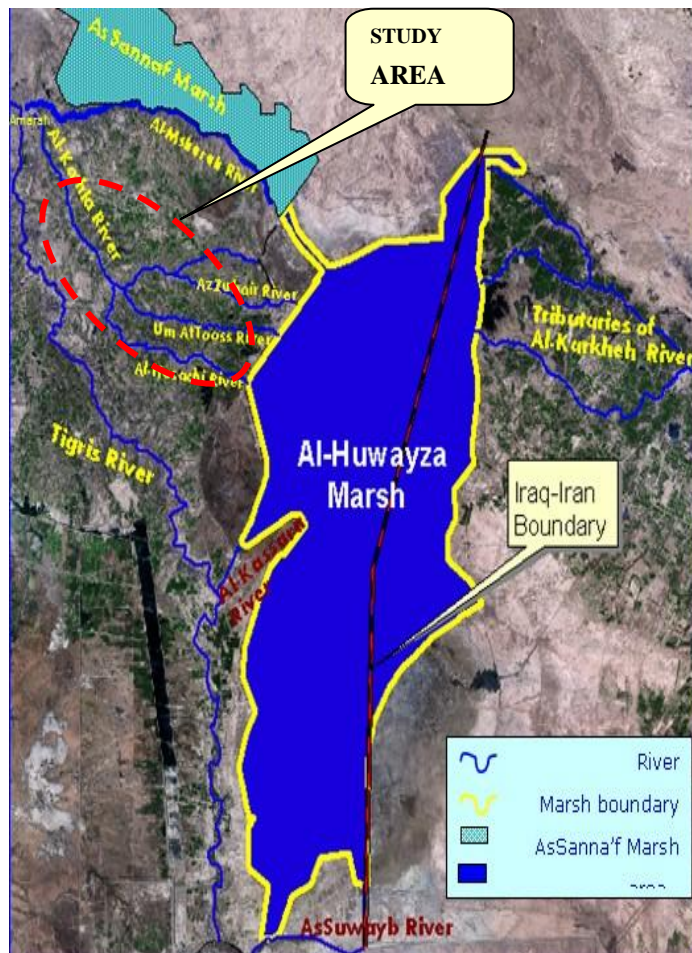


Figure (2) General Satellite Image of Al Huwayza Marsh feeders.

THE HEC-RAS MODEL .VI

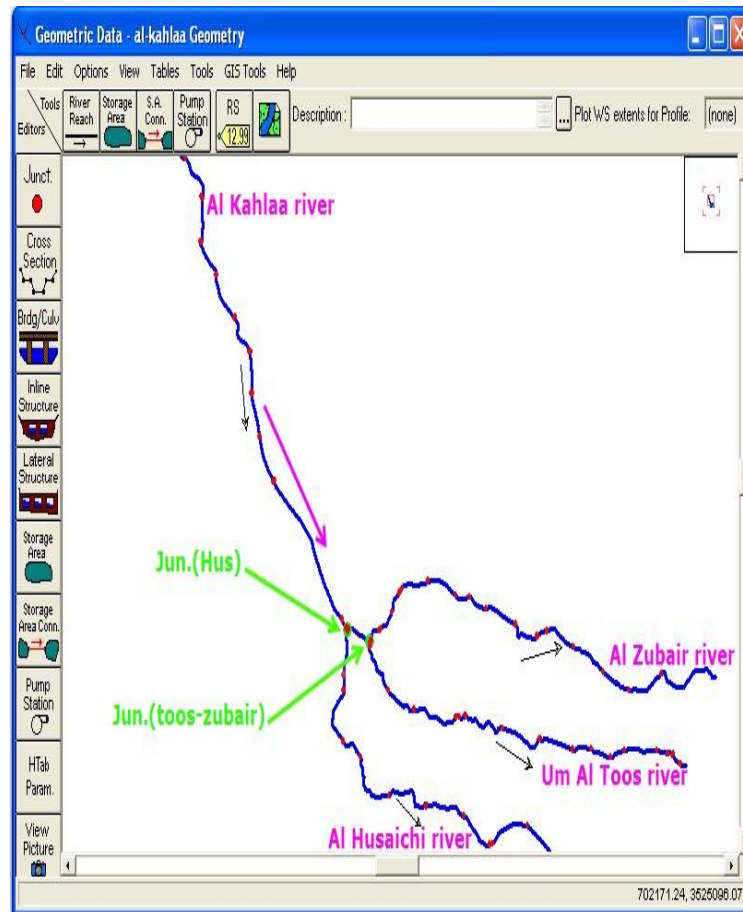
For this study, hydraulic model is developed using HEC-RAS [Version 4.1.0,(2009)]. The HEC-RAS model allows to perform simulation steady and unsteady flow evaluation in single or networked channels . It is an integrated system of software, designed for interactive use in multi-tasking, multi-user network environment. The system is comprised of a graphical user interface (GUI), separate hydraulic analysis components, data storage and management capabilities graphics and reporting facilities , HEC-RAS has been designed and

developed by the Hydrologic Engineering Center of the U.S. Army for 1D hydraulic calculations in natural and constructed channel systems [11].

For an unsteady flow model , two necessary file types are required :

- 1) The geometry file contains the necessary physical description of the stream reach ; and
- 2) The flow file describes the all flow inputs and related boundary conditions needed for the unsteady flow analyses .

All the input data required to run the Al- Kahlaa River HEC-RAS model are presented in the following sections.



6.1.THE GEOMETRIC DATA

The first step to develop HEC-RAS model is to create a HEC-RAS geometric file . The basic geometric data consist of establishing how the various river reaches are connected (River System Schematic), this schematic is developed by drawing and connecting the various reaches of the system within the geometric data editor . The schematic data must be the first input into the HECRAS model . **Figure 3** shows a schematic layout of the Al- Kahlaa system HEC-RAS model. The next step is to enter the necessary Al- Kahlaa system HEC-RAS model geometry are cross section data and reach lengths. Cross sections should be perpendicular to the anticipated flow lines and extend across the entire flood plain (these cross sections may be curved or bent) . The cross section is described by entering the station and elevations (x-y data) from left to right.

The measured distance between any two cross sections is referred to a reach length . All of the required information is displayed on the cross section data editor as shown in **Figure 4** , and also from **Table 1** to **Table 2** , demonstrate the importance of the cross sections geometry and locations on Al Kahlaa system, as well as to Basic Information for reach Length between these cross sections.

Figure (3) The AL-Kahlaa system schematic(Red point indicated the location of cross sections)

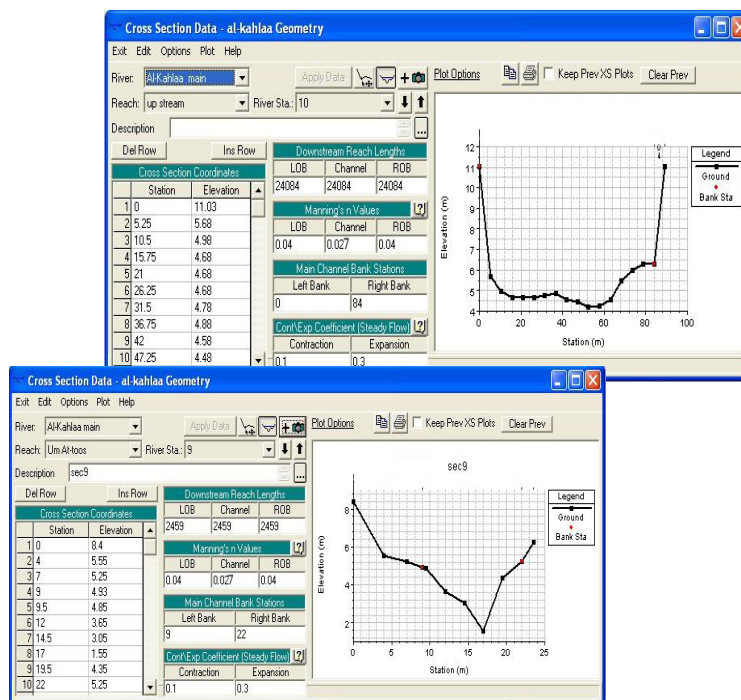


Figure (4). Input menu of cross section data

Table (1) The location of the cross-sections on (AL-Kahlaa , Al Zubair , Al Husaichi) river

River Name	Branch name	Cross section no.	Location		Channel Distance (m)
			Easting (m)	Northing (m)	
Al Kahlaa River	Main River	1	705386	3525335	4144
		2	706622	3522001	4200
		3	709070	3518971	3702
		4	710353	3515859	4020
		5	711979	3512320	4127
		6	714833	3509676	3891
		7	716906	3506688	463
		8	717271	3506389	100
		9	717323	3506325	86
		10	717342	3506327	874
		11	718200	3506135	800
	12	718961	3505819	170	
	Al Zubair River	1	719053	3505816	934
		2	719618	3506254	148
		3	719708	3506333	1171
		4	720531	3506976	1089
		5	721118	3507759	281
		6	721292	3507951	1981
		7	723215	3508230	2150
		8	725213	3507649	1096
		9	726243	3507568	1096
		10	727473	3507130	2085
11		729648	3506475	2129	

Al Husaichi River	12	731165	3505997	845
	Culvert	731180	3506001	
	13	731941	3506218	2116
	14	733874	3505589	2100
	15	736038	3504579	2078
	16	737919	3503827	2920
	17	740575	3504558	0
	1	717233	3506218	1925
	2	717004	3504387	617
	3	717001	3503780	2758
	4	716967	3501712	1569
	5	718243	3501096	3369
	6	720387	3499527	3268
	7	723087	3499161	2931
	8	725840	3498688	2622
	9	727905	3497550	2923
	10	730515	3498331	2565
11	732609	3496893	210	

Table (2) The location of the cross-sections on Um AlToos river

River Name	Branch name	Cross section no.	Location		Channel Distance (m)
			Easting (m)	Northing (m)	
Al Kahlaa River	Um AlToos River	1	718956	3505655	149
		2	718905	3505567	2408
		3	720259	3503781	2250
		4	722217	3502952	3366
		5	725277	3502711	240
		6	725472	3502711	504
		7	725952	3502753	3274
		8	728697	3502211	3163
		9	731395	3501791	2459
		10	733821	3501355	1412
		11	735160	3501406	1481
		12	736617	3501058	1105
		13	737633	3501339	2348
		14	739942	3501363	825
		15	740755	3501241	434
		16	741150	3501174	691
		17	741685	3500783	0

6.2. UNSTEADY FLOW DATA

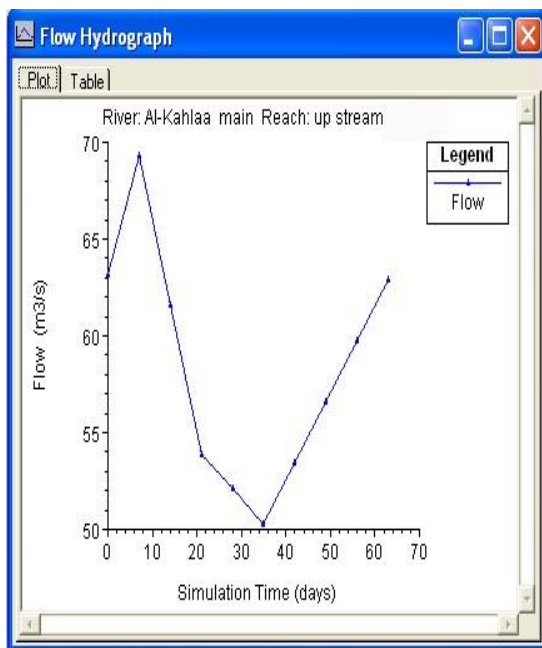
Unsteady flow data are required in order to perform an unsteady flow analysis. The unsteady flow data file consists of the boundary conditions and initial conditions for the model. The initial conditions contain the initial flow and stage profile for each cross section within the river network. The boundary conditions are the upstream and downstream boundary conditions defined as a stage hydrograph, or flow hydrograph .

VII. HEC-RAS CALIBRATION AND VERIFICATON

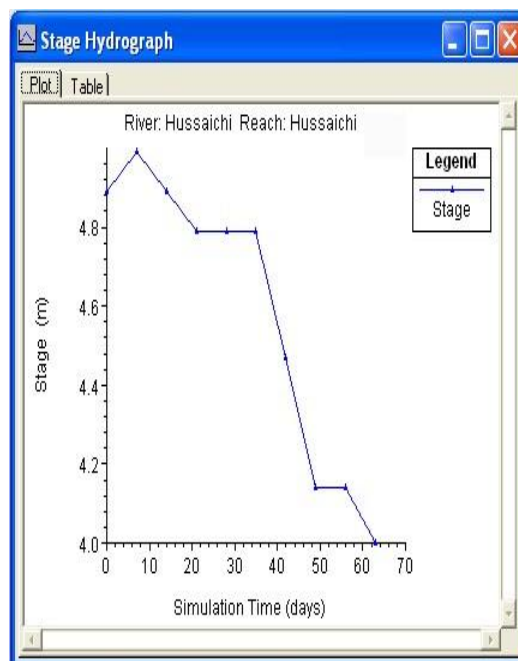
The HEC-RAS model calibration which is to document inputs and assumptions used in the development of Al- Kahlaa River HEC-RAS model to demonstrate that the model is adequate for use in evaluating the hydraulic models for several scenarios. This model is considered the primary goal to estimate the appropriate value of the Manning's roughness which is a key to making exact predictions.

For this study , it can be reasonably argued that the use of just two roughness in order to represent the heterogeneity of the region . Therefore, in the present study the Manning (n) may be selected via a trial-and-error calibration methodology . Al- Kahlaa River HEC-RAS model is repeated for different values of the

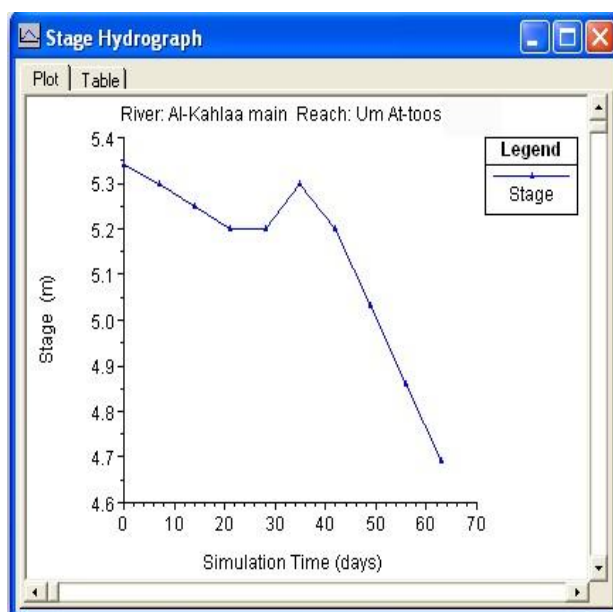
Manning (n) value . Best results are obtained when (n) is adjusted to reproduce observed data to an acceptable accuracy. With using observed flow hydrograph as the upstream boundary condition which is measured at the head regulator of Al- Kahlaa River as shown in **Figure 5** and select an observed stage hydrograph at the downstream end of the Um Al Toos and Al Hussaichi reach in most applications as shown in **Figure 6 to Figure 7** . The value of Manning’s n that resulted in the closest agreement between observed data and results of hydraulic simulation is determined by trial and error to be (n=0.04) for over bank with the values of Manning's (n=0.027) for main channel . The calibrated values then are used to verify the model predicted results with a set of data not used in the calibration.



Figure(5) Discharge hydrograph as upstream boundary condition



Figure(7) Stage hydrograph as downstream boundary condition of Al Hussaichi reach



Figure(6) Stage hydrograph as downstream boundary condition of Um Al Toos reach

VIII. APPLICATION AND RESULTS

For the applications of the current model to practical problems, some scenarios are reviewed or presented here. These scenarios cover a wide range of hydrological conditions, such as wet year and dry year. Consequently, the parameters used are based on the calibration results in the previous chapter. Brief descriptions are summarized in **Table 3**.

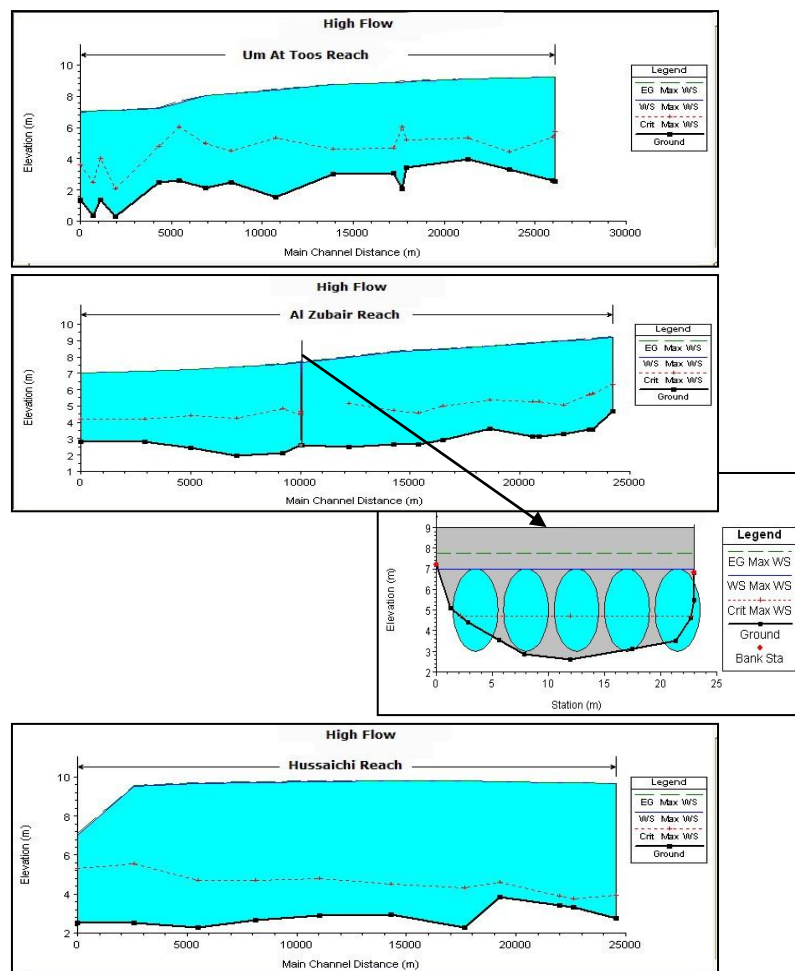
Table 3 Control Parameters

Parameter	Value
Finite Difference Weighing Factor (θ)	1
Time increment Δt	1 week
value of Manning for main channel	n=0.027
value of Manning for over bank	n=0.04

When the model was applied to simulate the high flows in wet year. The simulation results showed that most of the cross sections flooded and inadequate for such flows. While, flows remained within cross section extents when used the second scenarios with low flows.

It can clearly see that highest water level downstream Al Husaichi River are the same for Um Al Toos River and Al Zubair River was equal to 7 m a.m.s.l

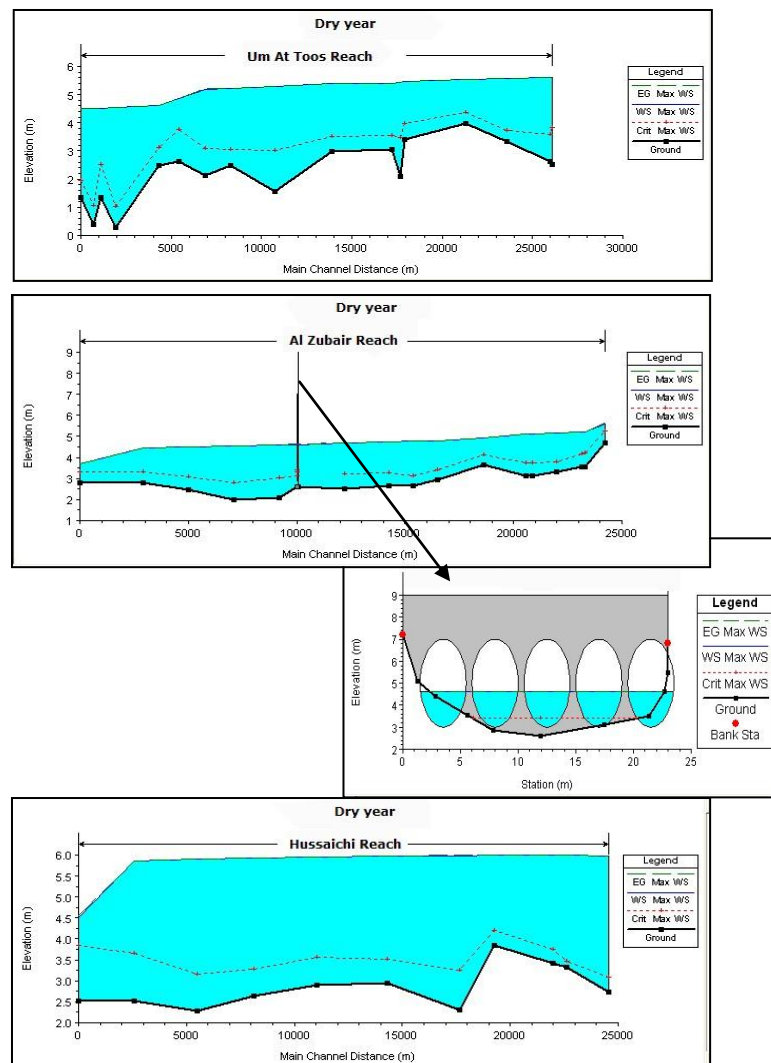
In drought season, the model is applied to the simulation of water level and flow during the drought period. It is clear that the lowest water levels approaches 3.7 m a.m.s.l at downstream Al Zubair River.



Figure(8) Water Surface Profile at High Flow

And also, know the total volume of water resulting from a flood its very important to assist in the design of storage facilities for flood control, irrigation and water supply. From **Figure 8** to **Figure 9** that demonstrates water surface profile at High Flow and Low Flow, respectively. It can see clearly the flow condition in every rivers is subcritical flow. From these profile the water surface appears higher than the

critical depth at several location .And also, these figures shows the water surface elevation decreased from upstream toward downstream in two cases because of water consumption and water losses along the study reach.



Figure(9) Water Surface Profile at Low Flow

IX. CONCLUSIONS

The HEC-RAS unsteady models was used for simulating flows and analyzing several scenarios of Al- Kahlaa river system . From the results of the model the following conclusions are drawn:

1. In simulation flows in a channel network, the accuracy of estimated roughness is a key to making exact predictions . This leads to the conclusion that (n) is best evaluated through calibration of the unsteady flow model before use , especially if reasonably accurate field data are available.

Results of analysis show that when applying the data of the case of high flow discharge of the Al- Kahlaa system , it is found that the water surface profile is higher than the longitudinal bank elevation everywhere along the reach for every river . In other words the Al- Kahlaa river system will be flooded in case of high flow, since its longitudinal bank elevation is lower than water surface. That means , cross sections is not sufficient for such high flows. Therefore , cross sections of each rive must be developed. Modifications to the channel cross-section may be from :

- This can be achieved by increasing the width or depth of the channel (thereby providing additional cross-sectional area for flow) . In some cases increased depth can be obtained by excavation, or by raising the height of the channel levees . Result from this channel modifications to accommodate a higher flow rate within the channel corridor

- channel dredging may temporarily provide additional flow area, subsequent sediment deposition will gradually reduce the channel conveyance. This is a common problem for most of the fluvial channels . In general, sedimentation has reduced the channel depth and width over time, resulting in reduced conveyance and increased flood hazards.

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